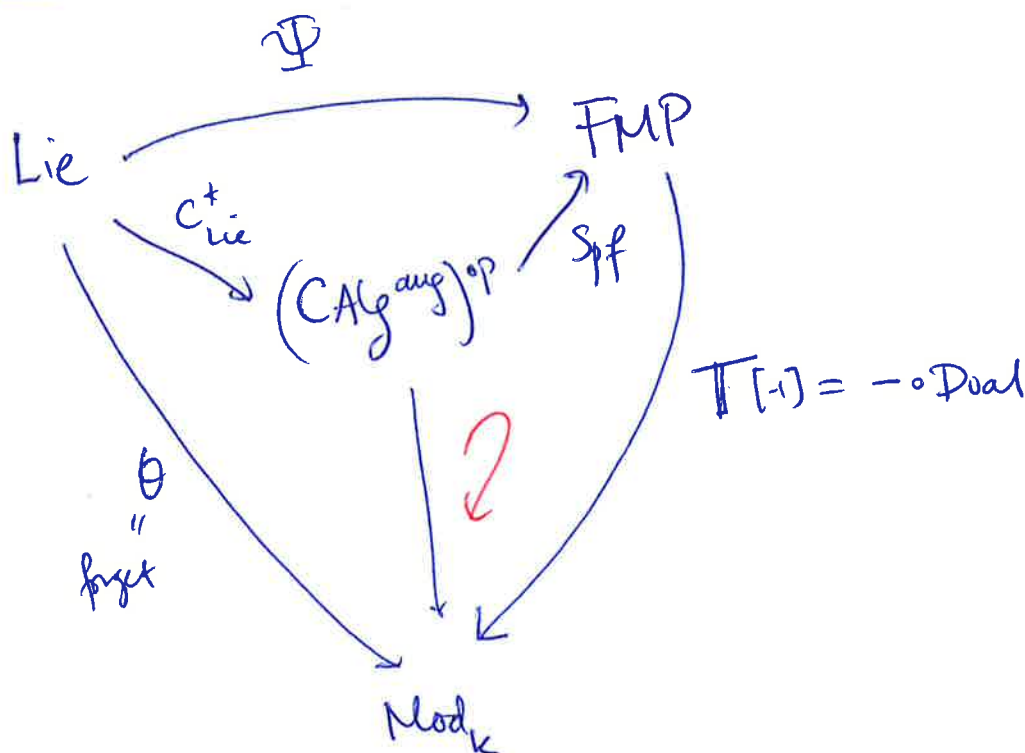


Part I: Intro

Today may be a bit disjointed.

We're trying to squeeze a lot of formalism about categories into one lecture so that we can do more concrete stuff toward the main theorem.

The current situation:



Want: $\mathcal{D}: (CAg^{aug})^{op} \rightarrow Lie$

$\Psi^{-1}: FMP \rightarrow Lie$

Q: How to show Ψ is an equivalence?

Idea: Find an "inverse"!

Standard strategy

Given $F: \mathcal{C} \rightarrow \mathcal{D}$, then

① find an adjoint $G: \mathcal{D} \rightarrow \mathcal{C}$

Recall one useful definition

Def A functor $F: \mathcal{C} \rightarrow \mathcal{D}$ is left adjoint to a functor $G: \mathcal{D} \rightarrow \mathcal{C}$ if there exists a natural transformation

$$\eta: \text{id}_{\mathcal{C}} \Rightarrow G \circ F \quad \text{["unit of adjunction"]}$$

such that for any $c \in \mathcal{C}$, $d \in \mathcal{D}$, and $f: c \rightarrow G(d)$ in \mathcal{C} , there exists a unique $g: F(c) \rightarrow d$ in \mathcal{D} such that

$$f = G(g) \circ \eta(c)$$

$$\begin{array}{ccc} \mathcal{C} & \xrightarrow{\eta(c)} & G(F(c)) & \xrightarrow{G(g)} & G(d) \\ & & \parallel & & \\ & & f & & \end{array}$$

also exists a counit $\epsilon: F \circ G \Rightarrow \text{id}_{\mathcal{D}}$

②

② show that

$$\eta: id_E \Rightarrow G \circ F \quad \& \quad \varepsilon: F \circ G \Rightarrow id_D$$

are isomorphisms

Looking for an adjoint feels easier & sometimes is obvious, even if the fact that it's an equivalence is not.

It also gives you a useful test criterion to see if your idea is plausible.

Prop A left adjoint preserves all colimits.
A right adjoint preserves all limits.

The proof is fun. If you don't know it, look it up!

There's a powerful partial converse.

Adjoint functor theorem(s)

If a functor $G: \mathcal{D} \rightarrow \mathcal{C}$ preserves limits and satisfies [SMALLNESS CONDITION],

then it has a left adjoint.

↑ several variants

③

So the rest of the talk will be devoted to:

① Learning more about colimits so we can prove (later!)

Prop C_{loc}^* preserves colimits.

You analyze a small list of kinds of colimits that generate them all

② Learning what an adjunction of ∞ -categories is & what "smallness condition" there provides an ∞ -categorical adjoint functor theorem